JAY-003-1163002 Seat No. ________

M. Sc. (Mathematics) (Sem. III) (CBCS) Examination

December – 2019

Maths : Functional Analysis : CMT - 3002

(Old & New Course)

Faculty Code : 003
Subject Code : 1163002

Time : $2\frac{1}{2}$ Hours] [Total Marks : **70**

Instructions :

(1) There are five questions.
(2) All questions are compulsory.
(3) Each question carries 14 marks.

1 Answer the following questions : (Any seven) **7×2=14**

1. Define :
   i) Taxicab metric
   ii) Accumulation point in a metric space.
2. Write in brief about the function space sequence space s.
3. Is every metric in a metric space complete? Justify.
4. Prove that every convergent sequence in a metic space is a Cauchy sequence.
5. Define :
   i) Cauchy sequence in a metric space
   ii) Cauchy sequence in a normed linear space.
6. Define Complete metric space with example.
7. State and prove Translation invariance of metric d in a metric space?
8. Define :
   i) Dual Space
   ii) Algebraic Dual Space.
9. Define :
   i) Linear Functional
   ii) Bounded Linear Operator.
10. Define Banach space with example.

2 Answer the following questions : (Any two) **2×7=14**

1. State and prove Holder inequality for sums.
2. State and prove Minkowski inequality for sums.
3. Prove that A subspace M of a complete metric space X is itself complete if and only id the set M is closed in X.
4. Prove the completeness of the space IR".

3 Answer the following questions : **2×7=14**

a) Let X = (X,d) be a metric space. Then, prove that a convergent sequence in X is bounded and its limit is unique.

b) Let X = (X, d) be a metric space. Then, prove that if $x_n \to x$ and $y_n \to y$, then $d(x_n, y_n) \to d(x, Y)$.

OR

a) Let T be a linear operaot. Then, prove that the range of T, R(T), is a vector space.

b) Let T be a linear operator. Then, prove that the null space of T, N(T), is a vector space.

4 Answer the following questions : (Any two) **2×7=14**

1. Prove that if in an inner product space, $x_n \to x$ and $y_n \to y$, then $<x_n, y_n> \to <x, y>$.
2. State and prove Parseval relation.
3. Let X and Y be the inner product spaces and $Q : X \to Y$ be a bounded linear operator. Then, prove that :
   a) Q = 0 if and only if $<Qx, y> = 0$ for all $x \in X$ and $y \in Y$.
   b) If $Q : X \to X$, where X is complex and $< Qx, x> = 0$ for all $x \in X$, then Q = 0.

5 Answer the following questions : (Any two) **2×7=14**

1. Define Hilbert space and Hilbert adjoint operator. Let H_1 and H_2 be the Hilbert spaces, $S : H_1 \to H_2$ and $T : H_1 \to H_2$ be the bounded linear operators and x be any scalar. Then, prove that

i) $< T^*y, x>$ _ $<y, Tx>$

ii) $(S + T)^* = S^* + T^*$.

iii) $(T^*)^* = T$.

iv) $T^*T = 0$ if and only if $T = 0$.

v) $(ST)^* = T^*S^*$.

2. Define Self ad joint operator and unitary operator. Let the operators $U : H \rightarrow H$ and $V : H \rightarrow H$ be unitary, where h is the Hilbert space. Then, prove that

i) U is isometric.

ii) $|| U || = 1$; provided $H \neq \{0\}$.

iii) $U^{-1} = (U^*)$ is unitary.

iv) U is normal.

v) UV is unitary.

3. State and prove Hann Banach Theorem (Normed linear spaces).

4. State and prove Riesz lemma.

5. State and prove Cauchy Schwarz inequality for an inner product space.